motion of the jet head is independent of the behavior of the remainder and, hence, of the nature of the change in pressure in the fuel system, except for the earliest stage of the process.

At higher pressures in the medium (7-10 atm or more) the transmission of information along the jet becomes reliable and the rear parts of the jet begin to affect the advance of the head. In this sense we use the term "property of longitudinal elasticity." It should also be noted that at pressures in the medium >5-7 atm the development of large-scale inhomogeneities dividing the jet into sections is not observed.

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LOCALIZED THERMAL STRUCTURE IN MEDIUM WITH BULK HEAT ABSORPTION

L. K. Martinson

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Investigations of nonlinear processes of the diffusion type [1-6] have revealed several qualitatively new features of the course of such processes in comparison with linear processes.

In particular, in heat-conduction processes nonlinearity can be responsible for such an unusual property as thermal inertia. In the wide sense the property of thermal inertia means a finite velocity of propagation of thermal perturbations, when the perturbations propagate in a nonlinear medium in the form of heat waves with a finite velocity of motion of the front.

The property of thermal inertia is manifested in a qualitatively new form when the thermal perturbations are spatially localized. In this case the front of the thermal perturbation, propagating from the source with finite velocity, penetrates only a finite depth into the medium even in an infinite period of time. As was shown in [7-10] the nonlinear spatial localization of thermal perturbations can be due to the effect of bulk absorption of thermal energy, the rate of which depends on the temperature.

One of the most interesting regimes of spatial localization of thermal perturbations is the stable localization regime [11]. The heat wave front in this localization regime remains stationary, and the size of the perturbation region does not vary with time. In this case the localized heat pulse is self-insulated from the surrounding space and evolves into a space region of constant size. As an example of realization of this type of nonlinear heat conduction regime we consider the evolution of a heat pulse in a medium of constant density ρ, heat capacity c, and thermal conductivity k in the presence of bulk absorption of thermal energy in it, the rate of which is related to the temperature by a power law and depends explicitly on the time – it decreases exponentially with time with a characteristic relaxation time τ. In the one-dimensional case this process is represented by the following parabolic quasilinear equation with a nonlinear lowest term:

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$$\rho c\, \partial u/\partial t = k\, \partial^2 u/\partial x^2 - q_0 u^\nu \exp(-t/\tau). \tag{1}$$

Here $q_0 = \text{const} > 0$, and $0 < \nu < 1$.

We will assume that the initial temperature distribution corresponding to the thermal structure of a solitary heat pulse is represented by a nonnegative continuous function $u_0(x)$, which vanishes outside the interval $(-l, +l)$, where $l < +\infty$. As the characteristic temperature U of the problem we select the maximum temperature of the thermal structure at the initial time.

Introducing the dimensionless quantities

$$u' = u/U,\ x' = x/l,\ t' = at/l^2,$$
$$T = a\tau/l^2,\ \Pi = q_0 l^2/kU^{1-\nu}\ (a = k/\rho c) \tag{2}$$

and omitting the dashes from the dimensionless quantities, we rewrite Eq. (1) in the form

$$\partial u/\partial t = \partial^2 u/\partial x^2 - \Pi u^\nu \exp(-t/T),\ 0 < \nu < 1. \tag{3}$$

Let the parameters of the heat pulse at the initial time be such that

$$T = \frac{1-\nu}{\pi^2} \text{ and } \Pi = \frac{\pi^2 (1+\nu)}{2(1-\nu)^2}.$$

We will seek the solution of Eq. (3) in this case as a solution with separable variables

$$u(t, x) = \varphi(t)\psi(x), \tag{4}$$

and select the relation $\varphi(t)$ in the form

$$\varphi(t) = \left[\frac{\pi^2(1+\nu)}{2(1-\nu)^2}\right]^{\frac{1}{1-\nu}} \exp\left(-\frac{\pi^2 t}{(1-\nu)^2}\right). \tag{5}$$

Then, substituting (4), taking (5) into account, in Eq. (3) we obtain for the coordinate function $\psi(x)$ the nonlinear differential equation

$$\frac{d^2\psi}{dx^2} - \psi^\nu + \frac{\pi^2}{(1-\nu)^2}\psi = 0. \tag{6}$$

Equation (6) can be integrated and its particular solution has the form of a function periodic in x

$$\tilde{\psi}(x) = \left[\frac{2(1-\nu)^2}{\pi^2(1+\nu)}\cos^2\frac{\pi x}{2}\right]^{\frac{1}{1-\nu}},$$

which vanishes at the points $x = x_n = \pm(2n+1)$, $n = 0, 1\ldots$. We note also that at these points $\tilde{\psi}'(x_n) = \tilde{\psi}''(x_n) = 0$. Taking this into account, and also the fact that Eq. (6) has a trivial zero solution, we write the generalized solution of Eq. (6) $\psi(x) \in C^2(R^1)$, joining at points $x = \pm 1$ two of these solutions:

$$\psi(x) = \begin{cases} \tilde{\psi}(x), & x \in (-1, +1), \\ 0, & x \in R^1 \setminus (-1, +1). \end{cases}$$

Returning to dimensional quantities and taking (2) into account we find finally that if the initial temperature distribution in the medium has the form

$$u_0(x) = \begin{cases} U\left[\cos^2\frac{\pi x}{2l}\right]^{\frac{1}{1-\nu}}, & x \in (-l, +l), \\ 0, & x \in R^1 \setminus (-l, +l), \end{cases} \tag{7}$$

while the size of this thermal structure l (the halfwidth of the heat pulse at the initial time) and its amplitude U are connected with the absorption parameters q_0, τ, and ν by the relations

$$l = \pi\sqrt{\frac{a\tau}{1-\nu}},\quad U = \left\{\frac{2q_0\tau(1-\nu)}{\rho c(1+\nu)}\right\}^{\frac{1}{1-\nu}},$$

the solution of Eq. (1) with initial distribution (7) has the form

$$u(t, x) = \begin{cases} U\exp\left[-\frac{t}{\tau(1-\nu)}\right]\left\{\cos^2\frac{\pi x}{2l}\right\}^{\frac{1}{1-\nu}}, & x \in (-l, +l), \\ 0, & x \in R^1 \setminus (-l, +l). \end{cases} \tag{8}$$

The solution (8) represents the evolution of a localized heat pulse (7) in a medium with bulk heat absorption. The nature of the evolution of such a heat pulse is unusual in that bulk heat absorption suppresses the heat

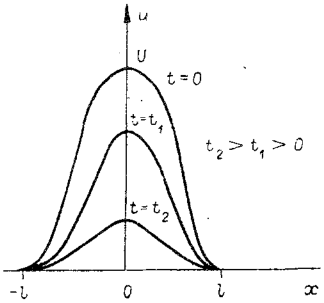

Fig. 1

diffusion mechanism and the heat pulse evolves on a zero unperturbed background without alteration of the width of the perturbation (region of space where $u > 0$). In other words, the carrier of solution (8) does not vary with time. The qualitative nature of the evolution of such a heat pulse is illustrated in Fig. 1.

We note that at the boundary points of the solution carrier $x = \pm l$, i.e., at the stationary frontal points of the heat pulse the physical conditions for continuity of the heat pulse are fulfilled at any instant. In addition, the solution (8) has all the derivatives, prescribed by Eq. (1), continuous throughout, i.e., in this sense it is a classical solution of Eq. (1). It is true that at points $x = \pm l$ the higher-order derivatives with respect to x may be discontinuous.

Solution (8) with a stationary heat front does not contradict the conclusion of classical heat-conduction theory – the velocity of propagation of thermal perturbation is infinite. This conclusion applies only to processes which are represented by a linear heat-conduction equation. Equation (1), however, is quasilinear and, as was shown in [12], can have solutions with a finite velocity of propagation of the thermal perturbations (in our case with zero velocity).

Thus, the obtained exact analytical solution of the quasilinear heat-conduction equation shows that in a medium with bulk heat absorption, whose specific power is related to the temperature by a power law, there may be thermal self-insulation of the structure, when the evolution of the thermal structure, owing to internal heat conduction mechanisms, proceeds without alteration of the spatial dimensions of the perturbation region.

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